

ANALYSIS OF SILICON STRESS/STRAIN  
RELATIONSHIPS

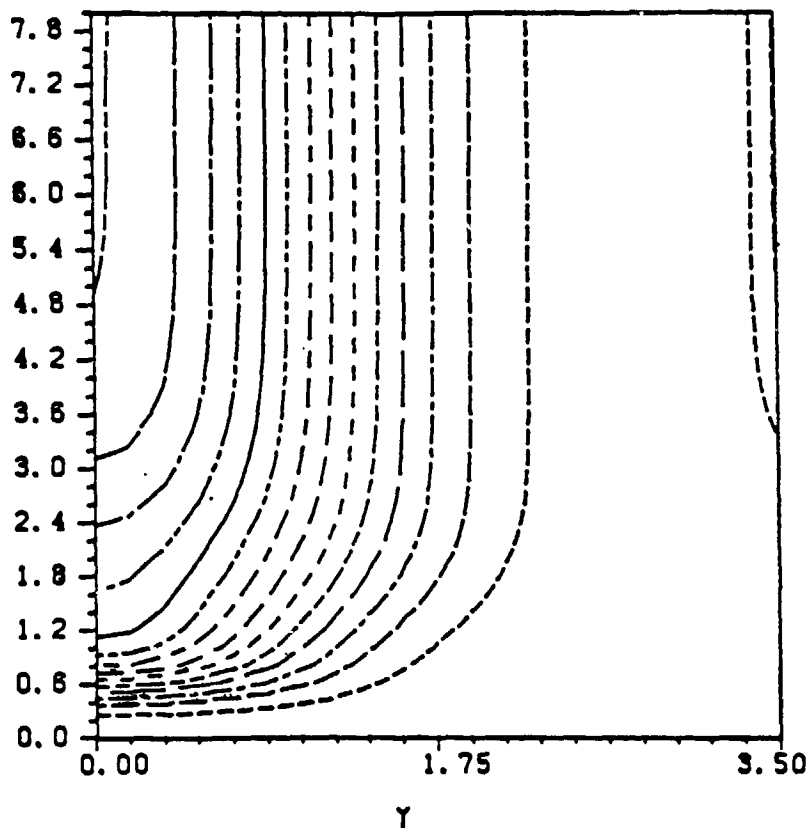
UNIVERSITY OF KENTUCKY

O. Dillon

## Dislocation Density Contour Plot

$$T = 1412 - 110.74X + 3.5X^2$$

UNIT OF X AND Y=CM. Z=1 PER CM=2



LEGEND: Z

-----	0.5	-----	9.6
-----	18.	-----	27.
-----	36.	-----	46.
-----	55.	-----	64.
-----	73.	-----	82.
-----	91.	-----	100.
-----	109.	-----	118.
-----	127.		

Dislocation density contour plot for the parabolic thermal profile of Eq.(4-8), 8x7 cm ribbon and an initial dislocation density =  $0.5 \text{ cm}^{-2}$ .

# ADVANCED SILICON SHEET

Dislocation Density Along  $Y = 0$  (Centerline)

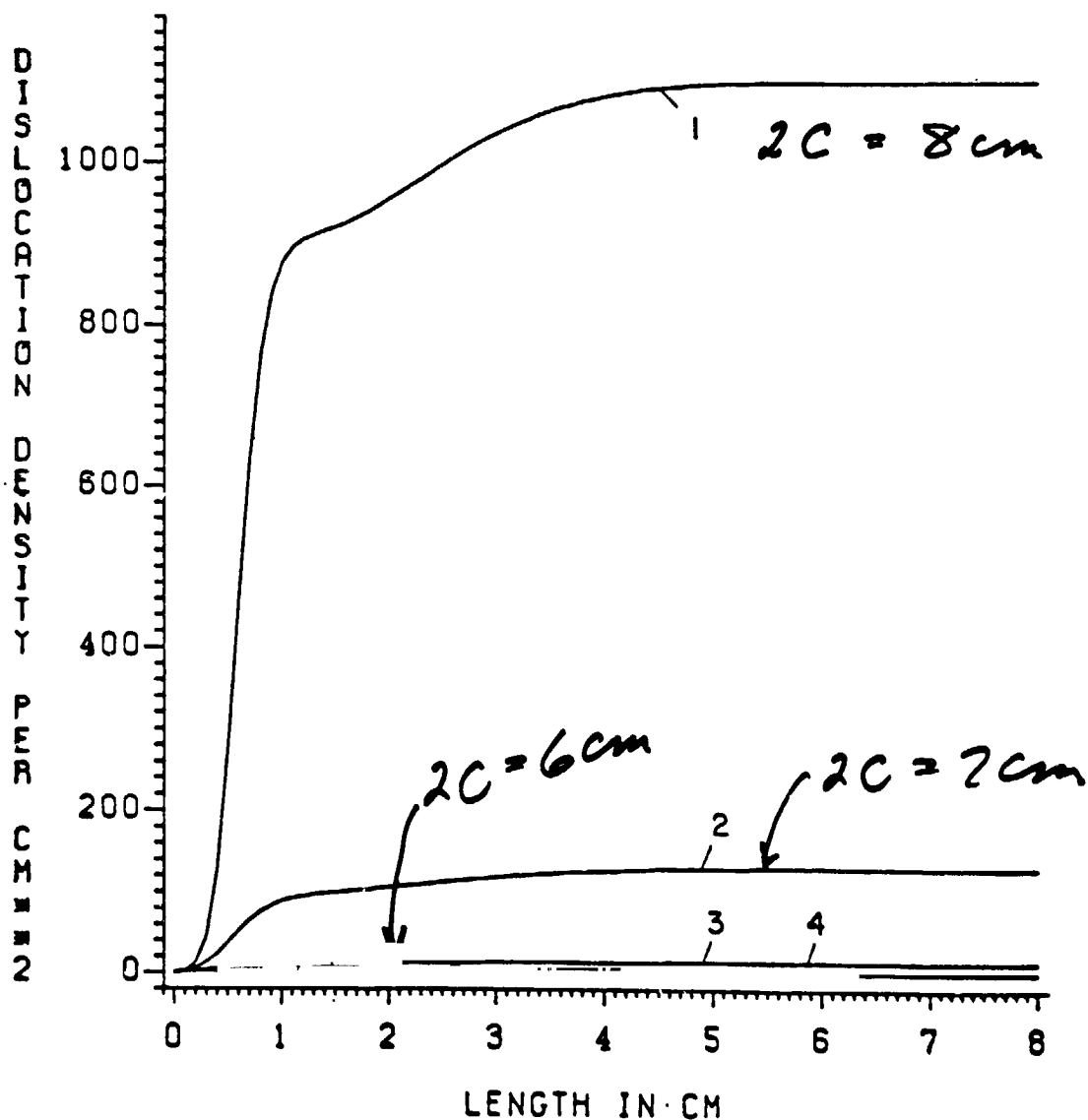
$$T = 1412 - 110.74 \times X + 3.5 \times X^2$$

LINE 1 FOR WIDTH=8 CM

LINE 2 FOR WIDTH=7 CM

LINE 3 FOR WIDTH=6 CM

LINE 4 FOR WIDTH=4 CM



Dislocation density along the centerline of the ribbon for the parabolic thermal profile of Eq. (4-8), initial dislocation density =  $0.5 \text{ cm}^{-2}$ , and width = 8, 7, 6, 4 cm.



# ADVANCED SILICON SHEET

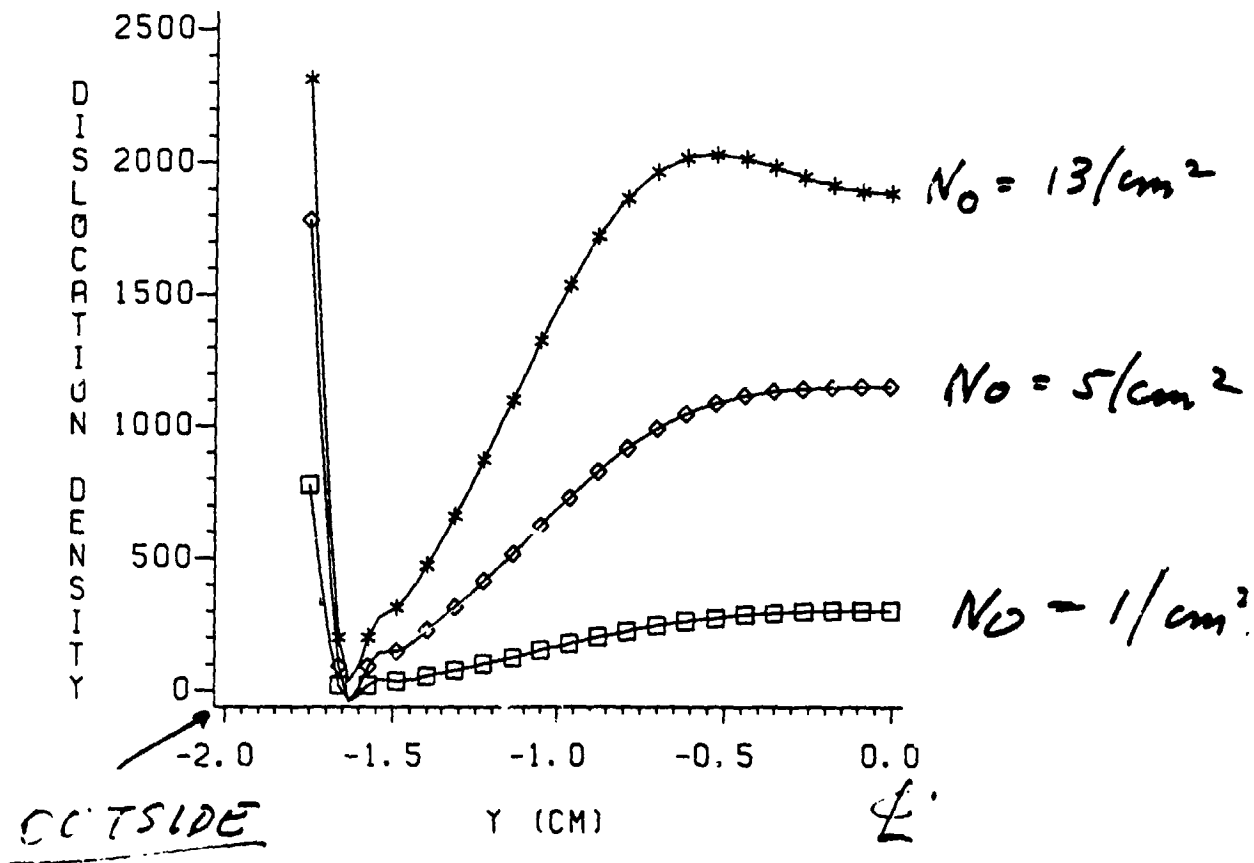
## Final Dislocation Density Along the Ribbon Width for Westinghouse Profile

LENGTH = 12 CM, WIDTH = 3.5 CM

STAR FOR  $N_0 = 13 / \text{CM}^2$

DIAMOND FOR  $N_0 = 5 / \text{CM}^2$

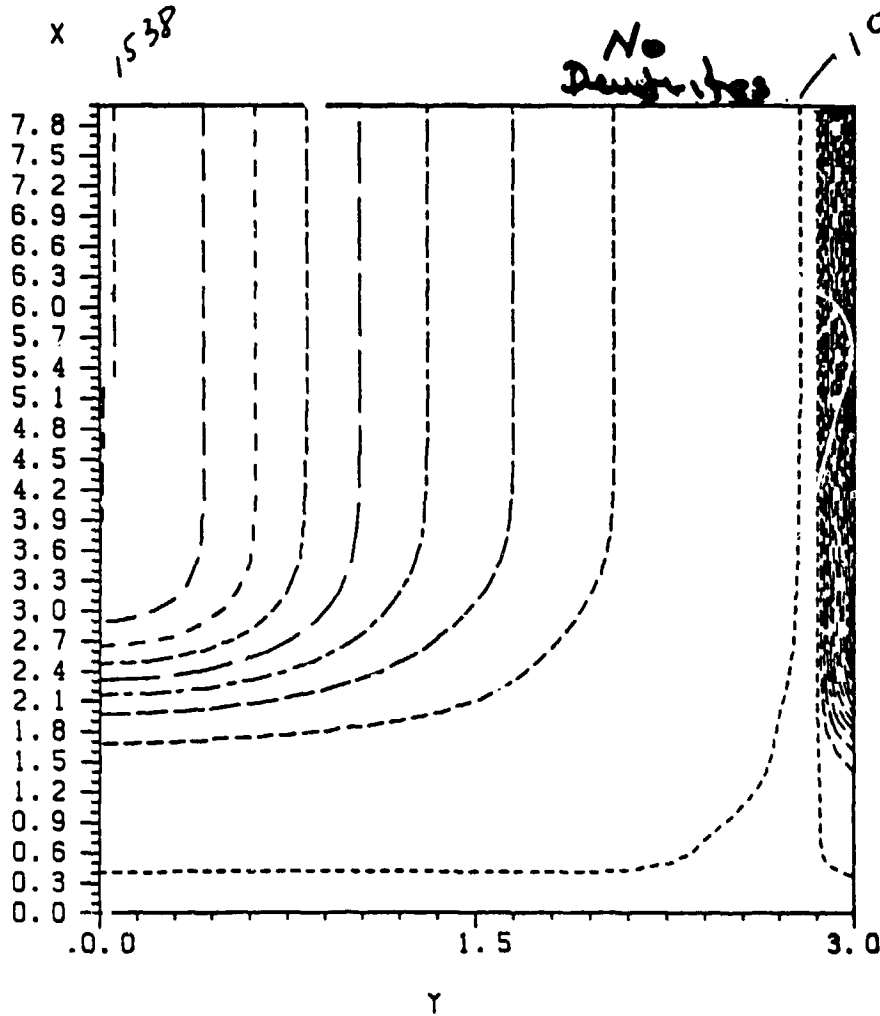
SQUARE FOR  $N_0 = 1 / \text{CM}^2$



# ADVANCED SILICON SHEET

## Dislocation Density Contour Plot

WESTINGHOUSE PROFILE, NO=0.25/CM<sup>2</sup>, R/T=0  
UNIT OF X AND Y=CM, Z=1 PER CM<sup>2</sup>



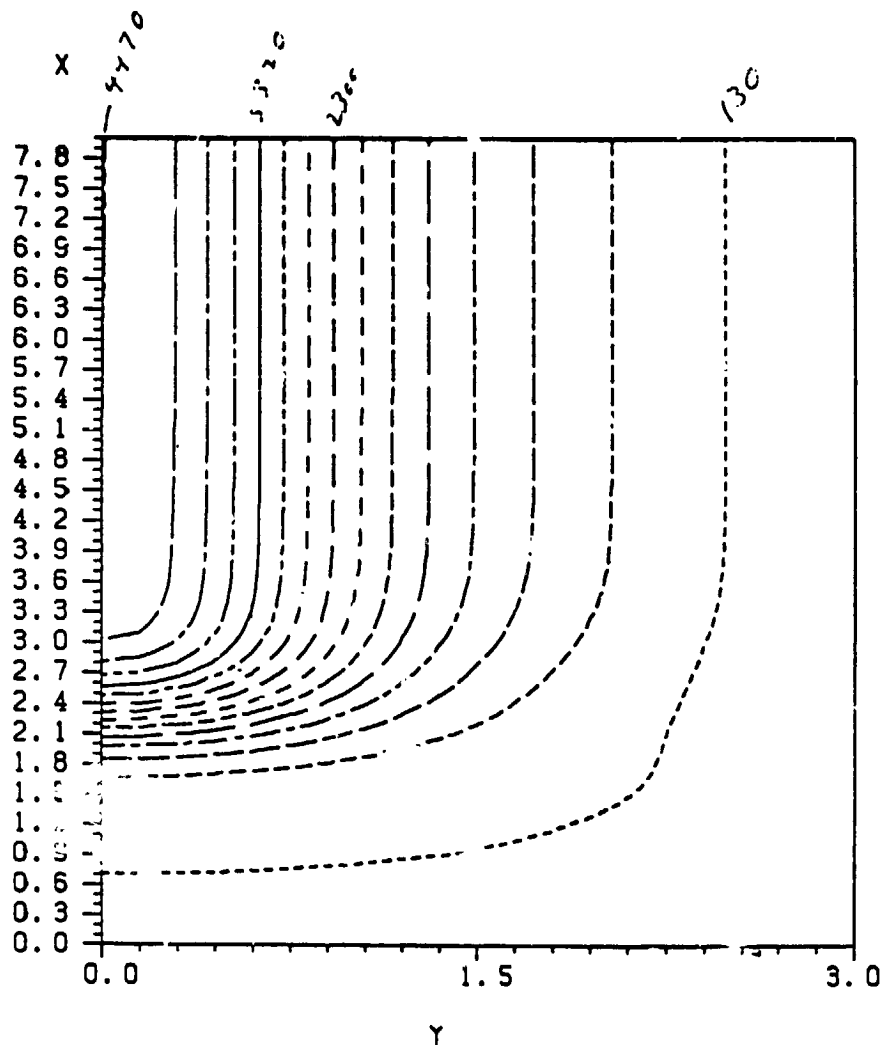
LEGEND: Z

-----	10	-----	201
-----	392	-----	583
-----	774	-----	965
-----	1156	-----	1347
-----	1538	-----	1729
-----	1920	-----	2111
-----	2302	-----	2493
-----	2684		

# ADVANCED SILICON SHEET

## Dislocation Density Contour Plot

WESTINGHOUSE PROFILE, NO=0.375/CM<sup>2</sup>  
UNIT OF X AND Y=CM, Z=1 /CM<sup>2</sup>, A/T=1.6667 M



LEGEND: Z

-----	130	-----	440
-----	750	-----	1060
-----	1370	-----	1680
-----	1990	-----	2300
-----	2610	-----	2920
-----	3230	-----	3540
-----	3850	-----	4160
-----	4470		

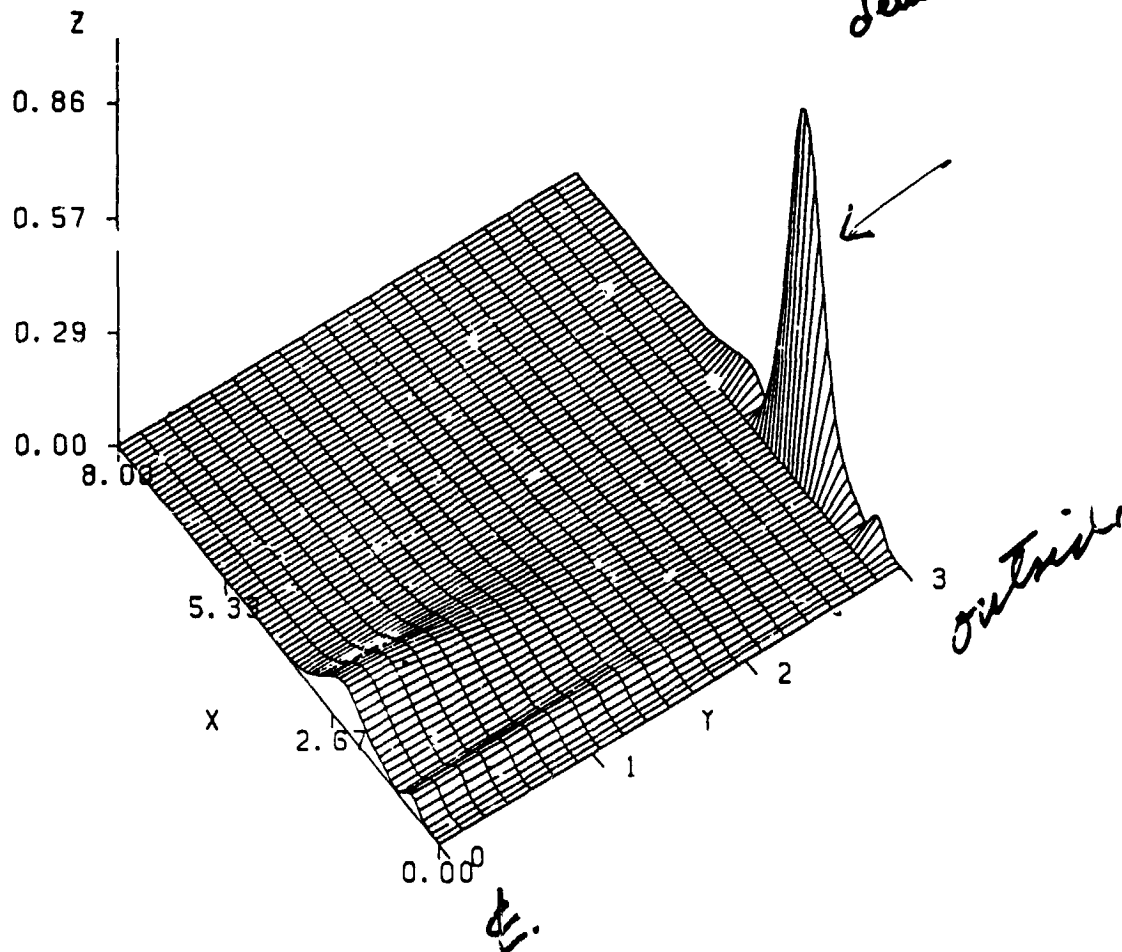
Note  
Low  
Densities

# ADVANCED SILICON SHEET

## Effective Plastic Strain Rate

WESTINGHOUSE PROFILE, NO=0.25/CM $\times$ 2  
WIDTH = 6 CM, LENGTH = 8 CM, A/T=0.  
UNIT OF X AND Y=CM, Z=10 $\times$ 10<sup>-5</sup> PER SEC

PLASTIC STRAIN  
RATE



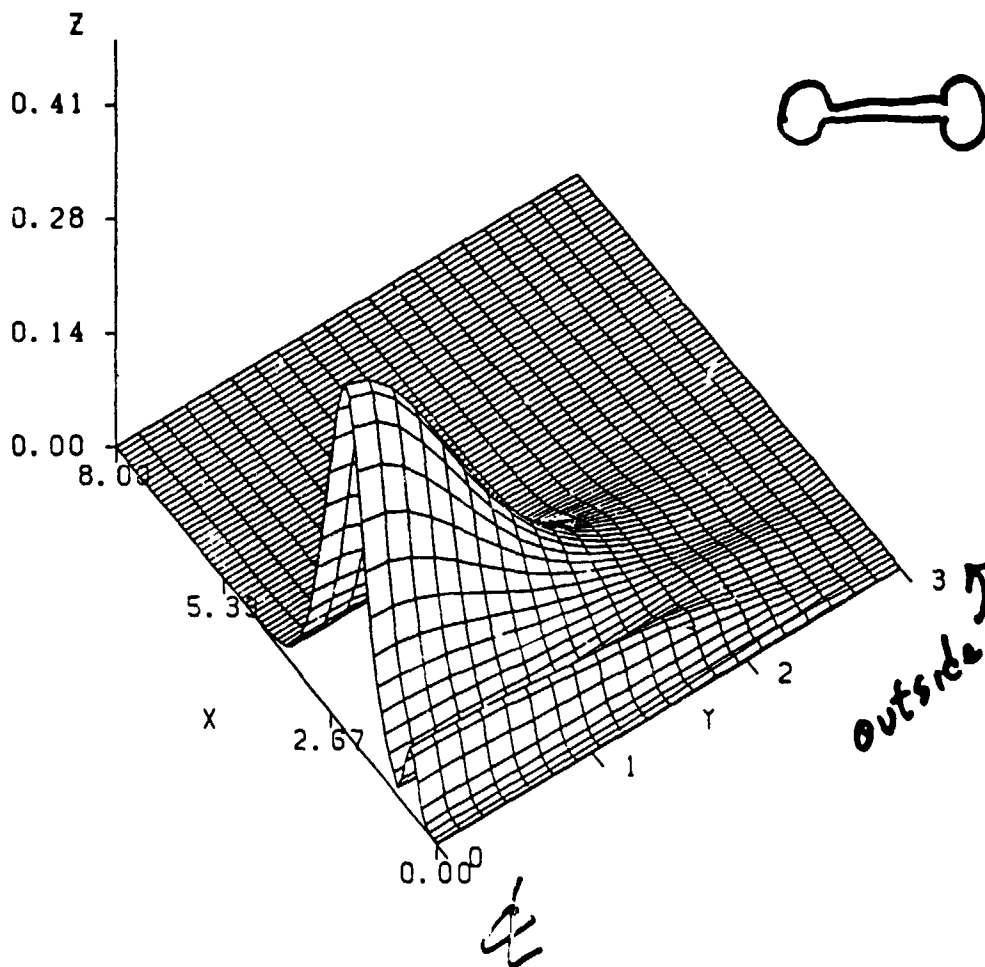
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# ADVANCED SILICON SHEET

## Effective Plastic Strain Rate

WESTINGHOUSE PROFILE, NO-0.375/CM $\times$ 2  
WIDTH = 6 CM, LENGTH = 8 CM, R/T=1.6667 MM  
UNIT OF X AND Y=CM Z=10 $\times$ 10<sup>-5</sup> PER SEC

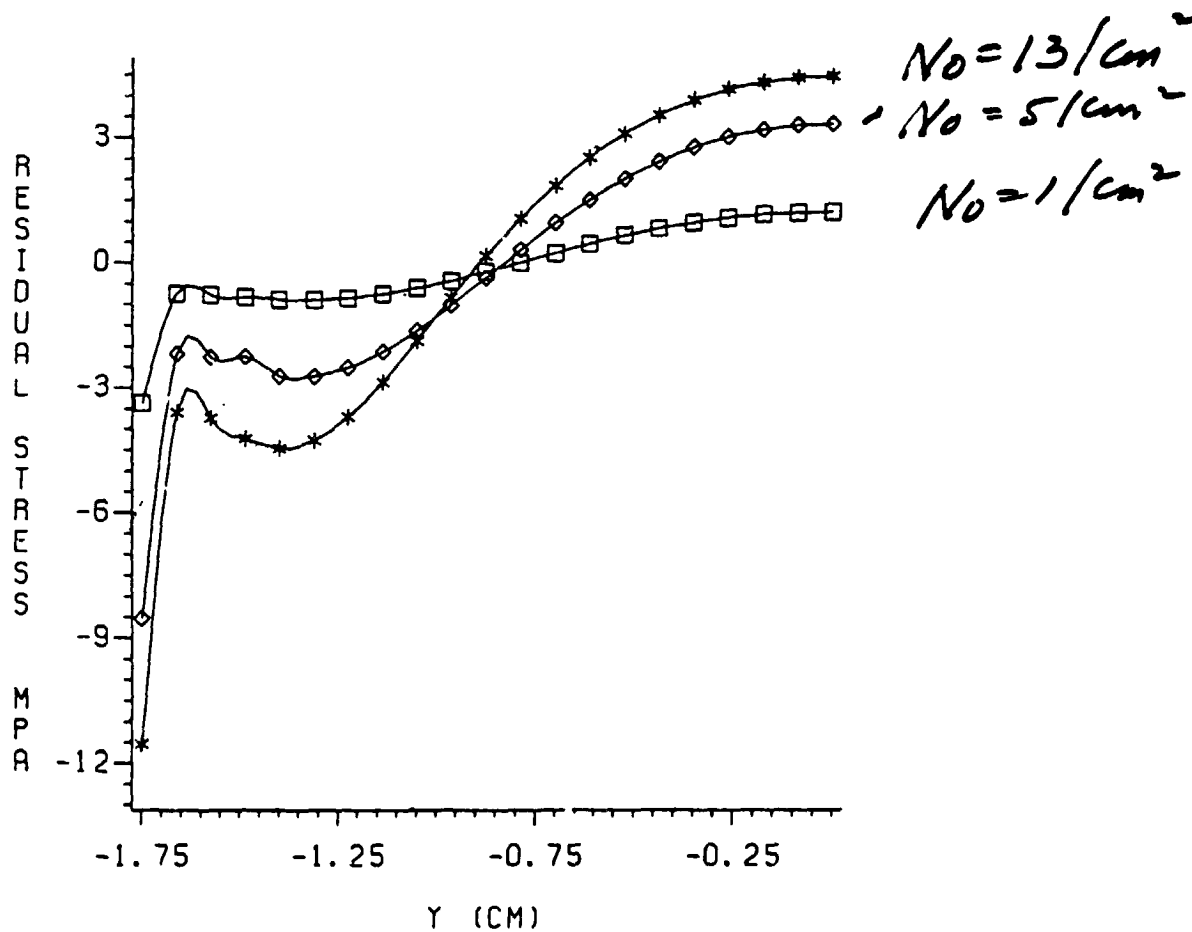
PLASTIC STRAIN RATE



ADVANCED SILICON SHEET

Residual Stress XX Along Ribbon  
Width for Westinghouse Profile

LENGTH = 12 CM. WIDTH = 3.5 CM  
STAR FOR NO = 13 /CM\*\*2  
DIAMOND FOR NO = 5 /CM\*\*2  
SQUARE FOR NO = 1 /CM\*\*2

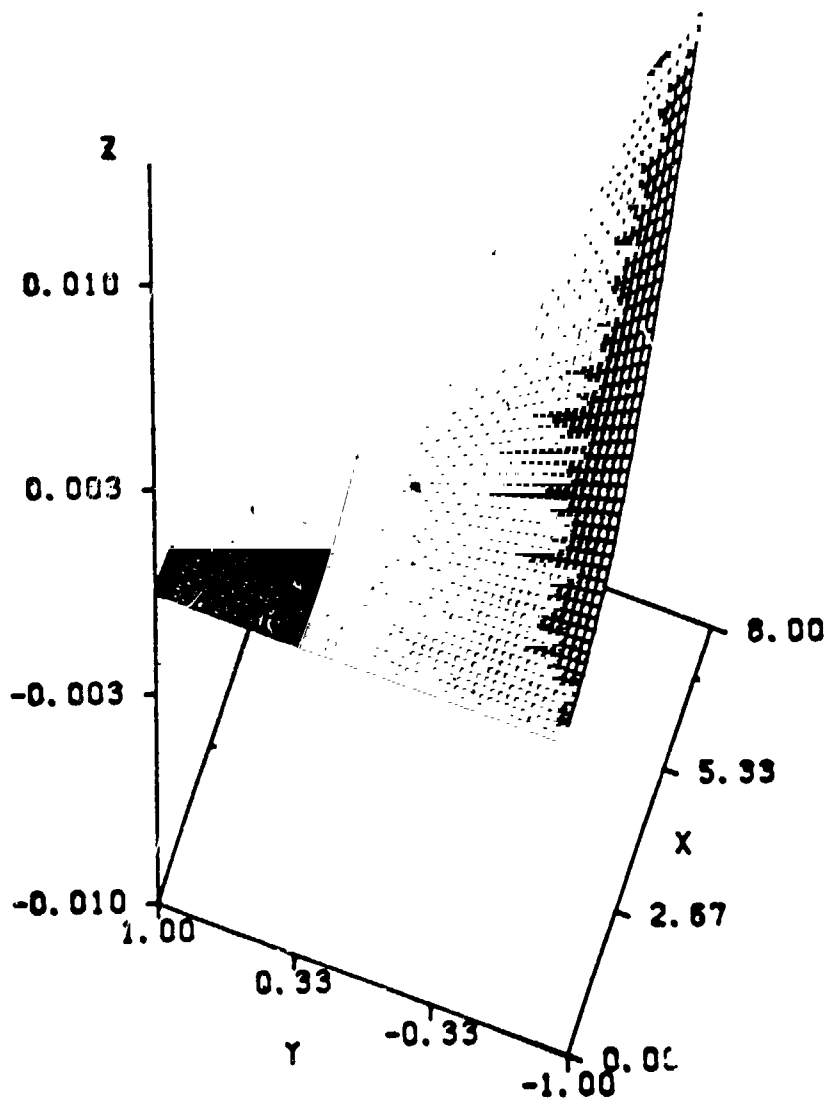




## Deflection Shape

HALF-WIDTH (C)=1.0 LENGTH=8.0  
DIAMETER OF DENDRITES IS 0.0 INCHES  
CRITICAL THICKNESS = 0.00526 INCHES

$$T(x) = \text{parabolic}$$

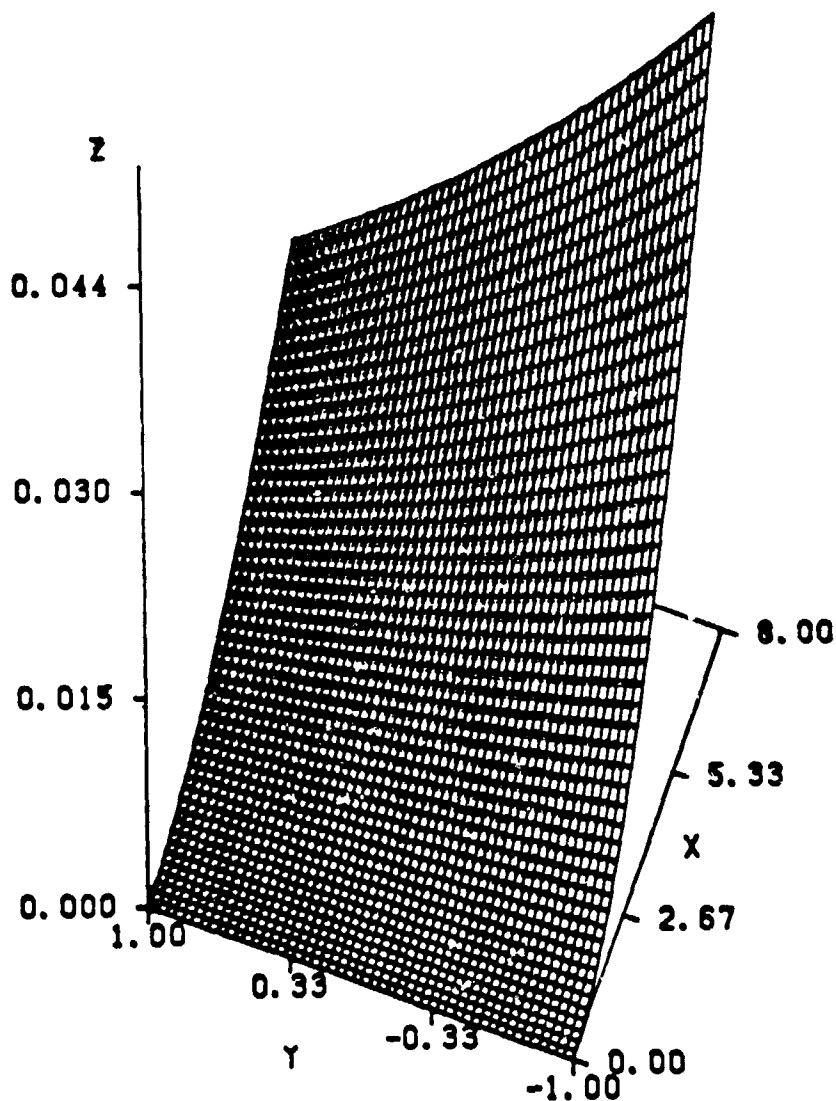


# ADVANCED SILICON SHEET

## Deflection Shape

HALF-WIDTH (C)=1., LENGTH=8.0  
DIAMETER OF DENDRITES IS 0.2 INCHES  
CRITICAL THICKNESS = 0.057120 INCHES

$$T(x) = \text{Parabolic}$$



Topics

## 1 Dislocation Motion

- A) Problem Formulation
- B) Calculation of Forces
- C) Tracking the motion of a single Dislocation

## 2 Dislocation Multiplication & Density

- A) Three methods of calculations - based on resolved shear stresses on each slip system
- B) Dislocation density by averaging the | shear stresses | in 0.5 cm. widths of ribbon; (starting at  $x = 0.2$  cm.)

## Possible Dislocations in the Silicon Crystal

burgers vector	tangent vector	slip plane type of dislocation
$\langle -1\ 0\ -1 \rangle$	$\langle -1\ 0\ -1 \rangle$	$(-1\ -1\ 1)$ , left, screw
=	$\langle 0\ 1\ 1 \rangle$	= ,left, $60^\circ$ $(-120^\circ)$ ✓
=	$\langle 1\ -1\ 0 \rangle$	= ,left, $60^\circ$ $(+120^\circ)$
=	$\langle 1\ -2\ -1 \rangle$	= ,left, edge
$\langle 0\ 1\ 1 \rangle$	$\langle 0\ 1\ 1 \rangle$	= ,left, screw
=	$\langle 1\ -1\ 0 \rangle$	= ,left, $60^\circ$ $(+120^\circ)$
=	$\langle -1\ 0\ -1 \rangle$	= ,left, $60^\circ$ $(-120^\circ)$ ✓
=	$\langle -2\ 1\ -1 \rangle$	= ,left, edge
$\langle -1\ 1\ 0 \rangle$	$\langle -1\ 1\ 0 \rangle$	= ,left, screw
=	$\langle 0\ -1\ -1 \rangle$	= ,left, $60^\circ$ $(-120^\circ)$ ✓
=	$\langle 1\ 0\ 1 \rangle$	= ,left, $60^\circ$ $(+120^\circ)$ ✓
=	$\langle -1\ -1\ 2 \rangle$	= ,left, edge
$\langle 1\ 0\ -1 \rangle$	$\langle 1\ 0\ -1 \rangle$	$(-1\ 1\ -1)$ , right, screw
=	$\langle -1\ -1\ 0 \rangle$	= ,right, $60^\circ$ $(-120^\circ)$ ✓
=	$\langle 0\ 1\ 1 \rangle$	= ,right, $60^\circ$ $(+120^\circ)$ ✓
=	$\langle -1\ -2\ -1 \rangle$	= ,right, edge
$\langle 0\ -1\ -1 \rangle$	$\langle 0\ -1\ -1 \rangle$	= ,right, screw
=	$\langle 1\ 1\ 0 \rangle$	= ,right, $60^\circ$ $(-120^\circ)$ ✓
=	$\langle -1\ 0\ 1 \rangle$	= ,right, $60^\circ$ $(+120^\circ)$
=	$\langle -2\ -1\ 1 \rangle$	= ,right, edge
$\langle 1\ 1\ 0 \rangle$	$\langle 1\ 1\ 0 \rangle$	= ,right, screw
=	$\langle -1\ 0\ 1 \rangle$	= ,right, $60^\circ$ $(+120^\circ)$
=	$\langle 0\ -1\ -1 \rangle$	= ,right, $60^\circ$ $(-120^\circ)$ ✓
=	$\langle -1\ 1\ -2 \rangle$	= ,right, edge
$\langle 0\ -1\ 1 \rangle$	$\langle 0\ -1\ 1 \rangle$	$(-1\ 1\ -1)$ , transv., screw
=	$\langle 1\ 1\ 0 \rangle$	= ,transv., $60^\circ$ $(+120^\circ)$ ✓
=	$\langle -1\ 0\ -1 \rangle$	= ,transv., $60^\circ$ $(-120^\circ)$ ✓
=	$\langle 2\ 1\ 1 \rangle$	= ,transv., edge
$\langle 1\ 1\ 0 \rangle$	$\langle 1\ 1\ 0 \rangle$	= ,transv., screw
=	$\langle 0\ -1\ 1 \rangle$	= ,transv., $60^\circ$ $(+120^\circ)$
=	$\langle -1\ 0\ -1 \rangle$	= ,transv., $60^\circ$ $(-120^\circ)$ ✓
=	$\langle 1\ -1\ 2 \rangle$	= ,transv., edge
$\langle 1\ 0\ 1 \rangle$	$\langle 1\ 0\ 1 \rangle$	= ,transv., screw
=	$\langle -1\ -1\ 0 \rangle$	= ,transv., $60^\circ$ $(-120^\circ)$ ✓
=	$\langle 0\ 1\ -1 \rangle$	= ,transv., $60^\circ$ $(+120^\circ)$
=	$\langle -1\ -2\ 1 \rangle$	= ,transv., edge

- (1). Surface of the ribbon is  $(1\ 1\ 1)$  plane.
- (2). Growth direction to the melt is  $\langle -2\ -1\ -1 \rangle$ .
- (3). For the motion of the dislocations,  $60^\circ$  dislocations that have  $-120$  degree with the burger's vector will be chosen because these  $60^\circ$  dislocations may multiply themselves more than  $+120^\circ$  type  $60^\circ$  dislocations as many investigators observed.

ORIGINAL PAGE IS  
OF POOR QUALITY

### Assumptions

- (1) The density of dislocation at the liquid solid interface is uniform
- ✱ (2) The pulling rate of the ribbon is 3cm/min.
- ✱ (3) Dislocations can move only in active slip systems that have their resolved shear stress higher than 95% of the most active slip system that has maximum Schmid Factor.
- (4) Average velocity of the dislocations in the presence of other dislocations is almost same as the velocity of isolated dislocation.  
Equivalent to low dislocation density.
- ✱ (5) The velocity equation proposed by K.Sumino

$$V = V_0 \tau \exp(-E/kT)$$

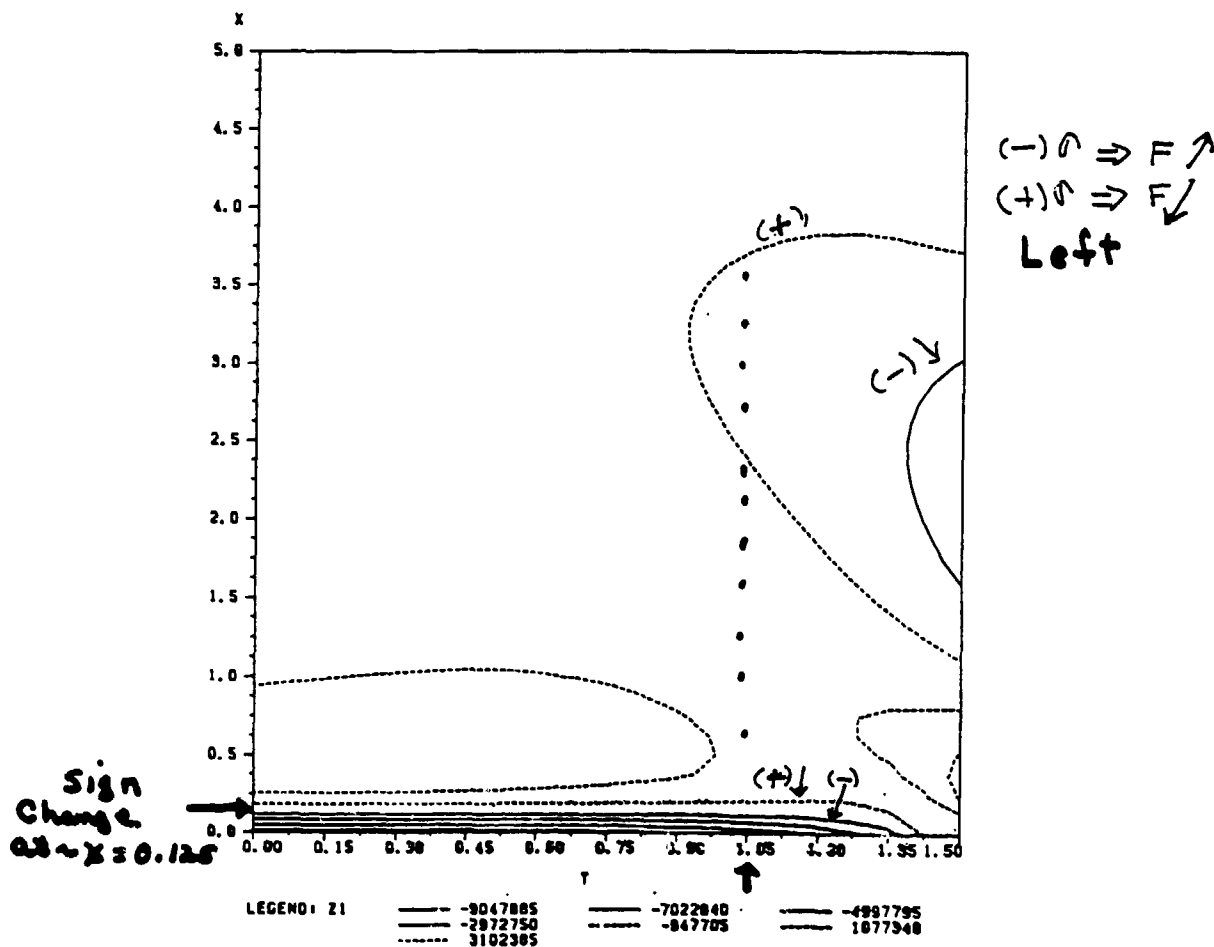
where E is 2.2 eV for 60' and 2.35 eV for screw,

$V_0$  is 0.035 for screw and 0.01 m<sup>3</sup> / MN.sec for 60'.

is still valid at high temperature like around melting temperature.

Resolved Shear Stress

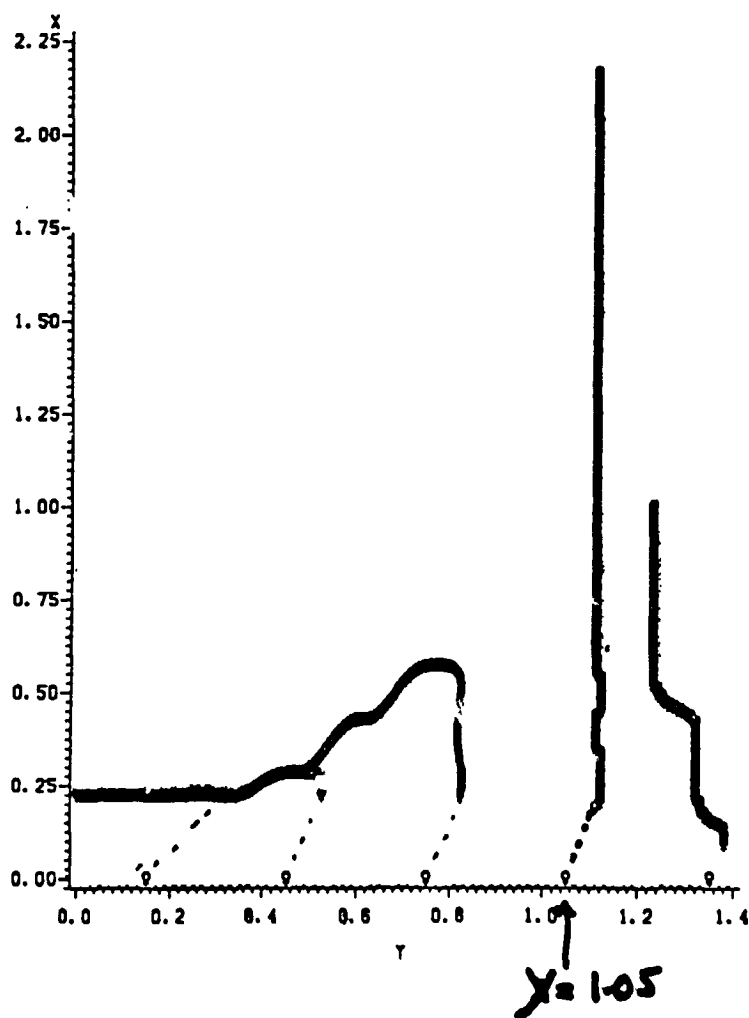
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ADVANCED SILICON SHEET

Motion of Dislocation

$B=R/2(-1 \ 1 \ 0)$ ,  $T=(1 \ 0 \ 1)$ ,  $M=(-1 \ -1 \ 1)$



UNIT OF AXES ARE CM

# ADVANCED SILICON SHEET

## Motion of the Dislocations Emerging to the Surface

possible 60° dislocations emerging to the surface

burger's vector	tangent vector	plane	motion
$a/2(-1\ 0\ -1)$	$(0\ 1\ 1)$	$(-1\ -1\ 1)$	left strong
$a/2(-1\ 1\ 0)$	$(0\ -1\ -1)$	$(-1\ -1\ 1)$	*
$a/2(-1\ 1\ 0)$	$(1\ 0\ 1)$	$(-1\ -1\ 1)$	split
$a/2(0\ 1\ 1)$	$(-1\ 0\ -1)$	$(-1\ -1\ 1)$	right weak
$a/2(1\ 0\ -1)$	$(0\ 1\ 1)$	$(-1\ 1\ -1)$	right strong
$a/2(1\ 0\ -1)$	$(-1\ -1\ 0)$	$(-1\ 1\ -1)$	*
$a/2(1\ 1\ 0)$	$(0\ -1\ -1)$	$(-1\ 1\ -1)$	*
$a/2(0\ -1\ -1)$	$(1\ 1\ 0)$	$(-1\ 1\ -1)$	right weak
$a/2(0\ -1\ 1)$	$(-1\ 0\ -1)$	$(1\ -1\ -1)$	right strong
$a/2(0\ -1\ 1)$	$(-1\ -1\ 0)$	$(1\ -1\ -1)$	left strong
$a/2(1\ 0\ 1)$	$(-1\ -1\ 0)$	$(1\ -1\ -1)$	left weak
$a/2(1\ 1\ 0)$	$(-1\ 0\ -1)$	$(1\ -1\ -1)$	left weak

\* ; These are forced into the liquid



## Calculation of the Density of Dislocations

From the K. Sumino's equation of dislocation multiplication

$$\Rightarrow dN_m = K K_0 N_{m1} (T_a - G b \sqrt{N_{m2}} / \beta)^{m+\lambda} \exp(-Q/kT) dt \quad \text{---(A)}$$

where  $K, K_0, b, \beta, m, \lambda, k, Q$  are constants given by K. Sumino

$N_m$ 's ; dislocation density

$N_{m1} \propto$  source density

$N_{m2}$  is the density controlling the back stress

$T_a$  ; applied stresses

$T$  ; temperature

$G$  ; shear modulus

$t$  ; time

Three possible ways of application of the equation (A)

(1)  $N_{m1}$  and  $N_{m2}$  are total density of dislocations

(2)  $N_{m1}$  is the partial density of dislocations on each slip system and

$N_{m2}$  is the total density of dislocations

(3) Both  $N_m$ 's are partial densities of dislocations on each slip system

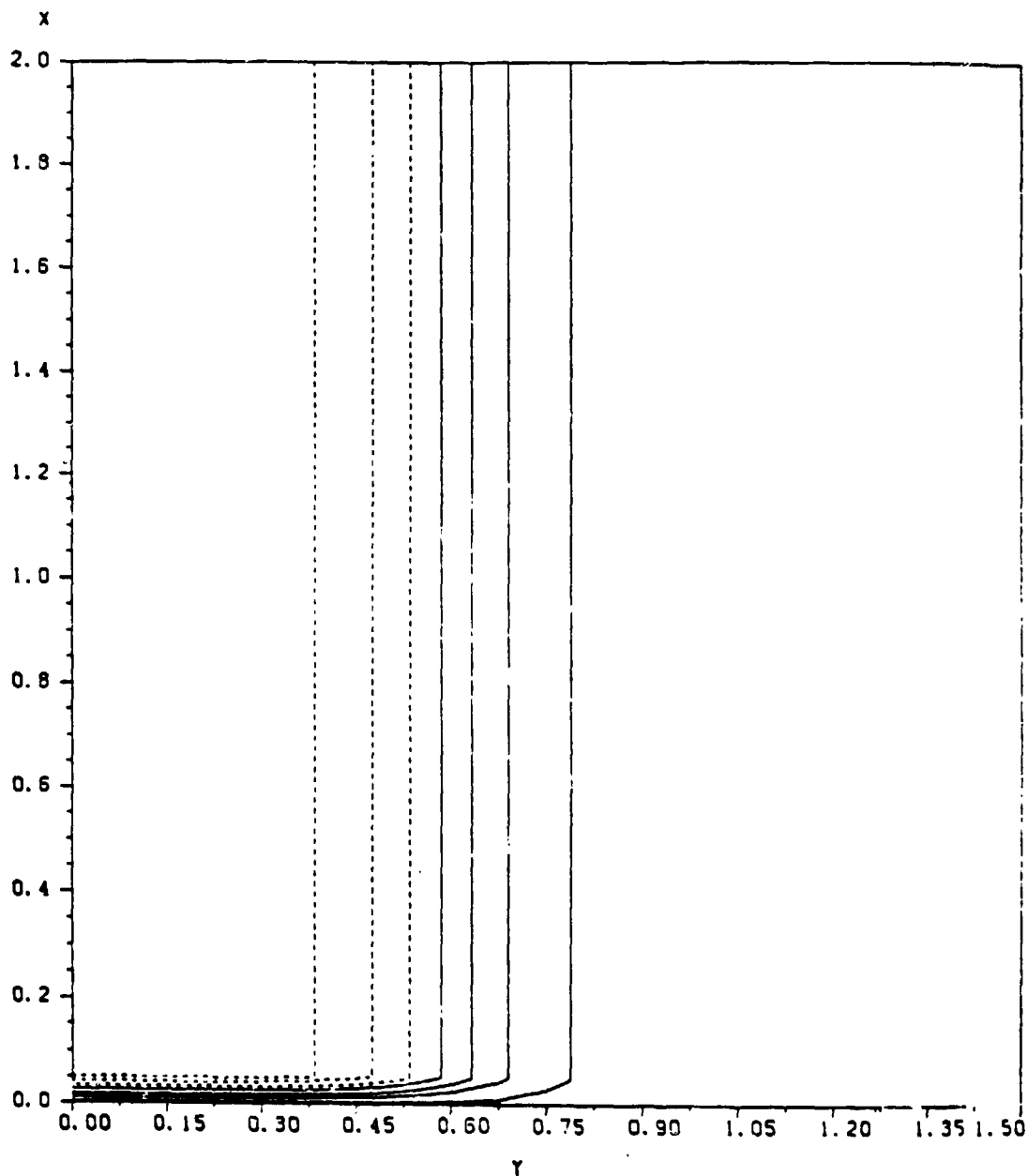
# Method Calculating Dislocation Density

- |     | $Nm_1$                    | $Nm_2$      |  |
|-----|---------------------------|-------------|--|
|     | -----                     | -----       |  |
| (1) | $\perp \rightarrow RSS_i$ | $\parallel$ | total density = $Nm_1 = Nm_2$<br>$dNm = dNm_1 = dNm_2$                             |
| (2) | $\perp \rightarrow RSS_i$ | $\parallel$ | total density = $\sum_i^g (Nm_1)_i = Nm_2$<br>$dNm_2 = \sum_i^g (dNm)_i$           |
| (3) | $\perp \rightarrow RSS_i$ | $\parallel$ | total density = $\sum_i^g (Nm_1)_i = \sum_i^g (Nm_2)_i$<br>$(dNm_1)_i = (dNm_2)_i$ |

# ADVANCED SILICON SHEET

## Total Density of Dislocation Using YI, YTOT and DGEAR

INITIAL TOTAL DENSITY IS 90/M\*\*2 LAMDA IS 1.0



LEGEND: DENS

—  
- - -  
.....  
.....

8.5915E+17  
6.0140E+18  
1.1169E+19  
1.6324E+19

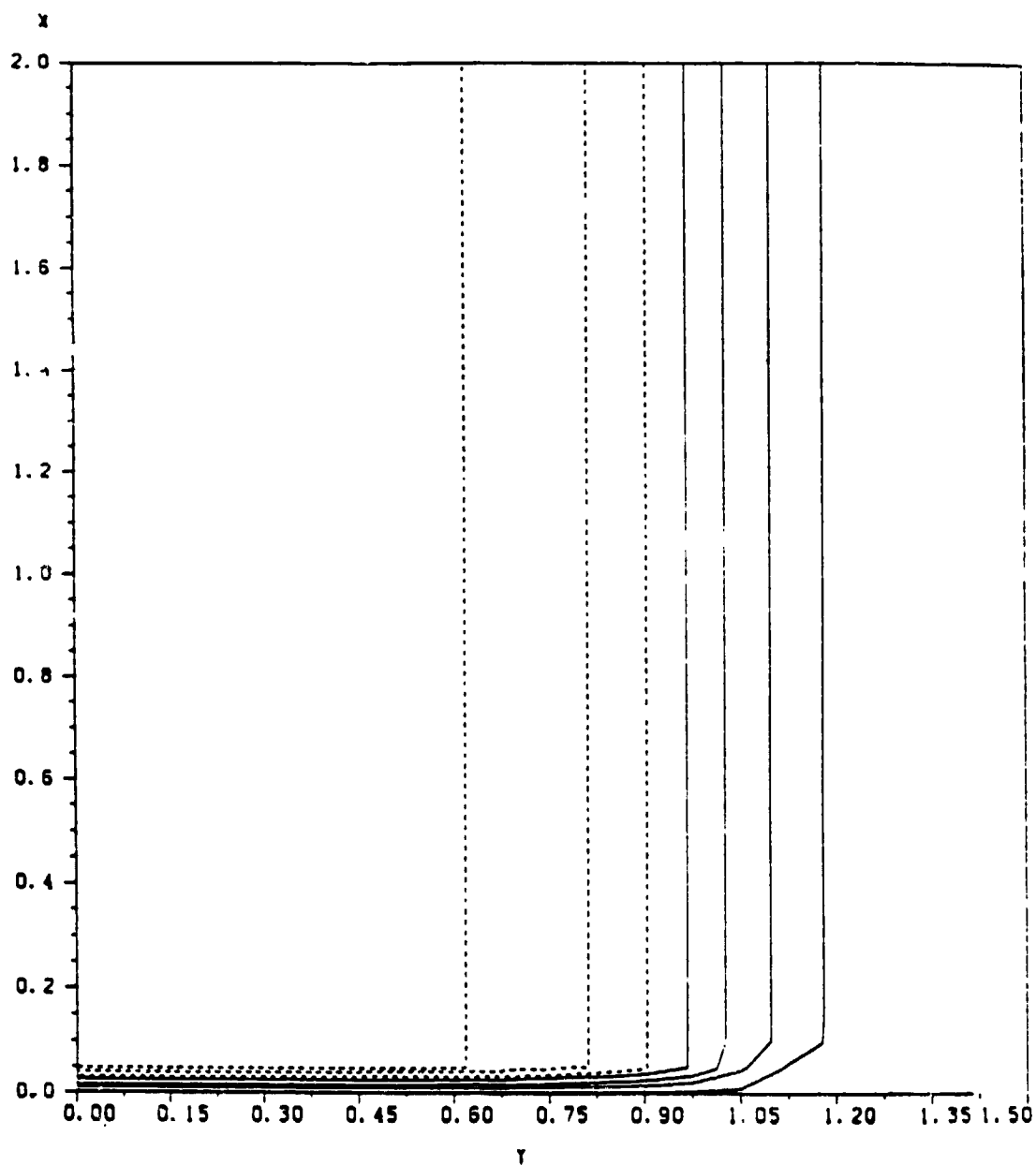
—  
- - -  
.....

3.4366E+18  
8.5915E+18  
1.3746E+19

# ADVANCED SILICON SHEET

## Total Density of Dislocation Using YI, YI and DGEAR

INITIAL TOTAL DENSITY IS  $90/M=2$  .LAMDA IS 1.0



LEGEND: DENS8

—  
- - -  
.....

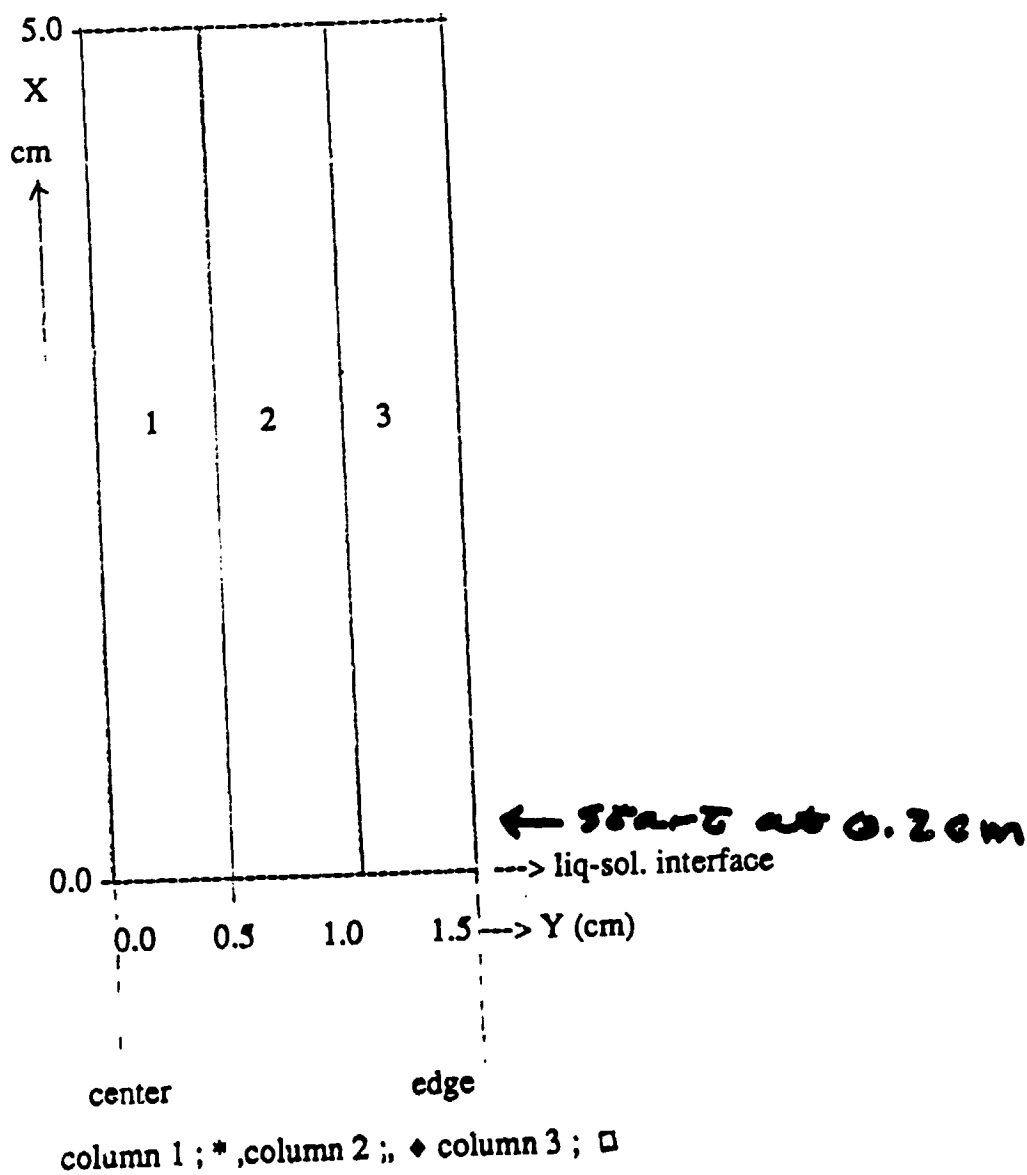
7.7220E+11  
5.4054E+12  
1.0090E+13

—  
- - -  
.....

3.0888E+12  
7.7220E+12  
1.0090E+13

# ADVANCED SILICON SHEET

Averaging the  $|\tau a|$  in Calculating Density



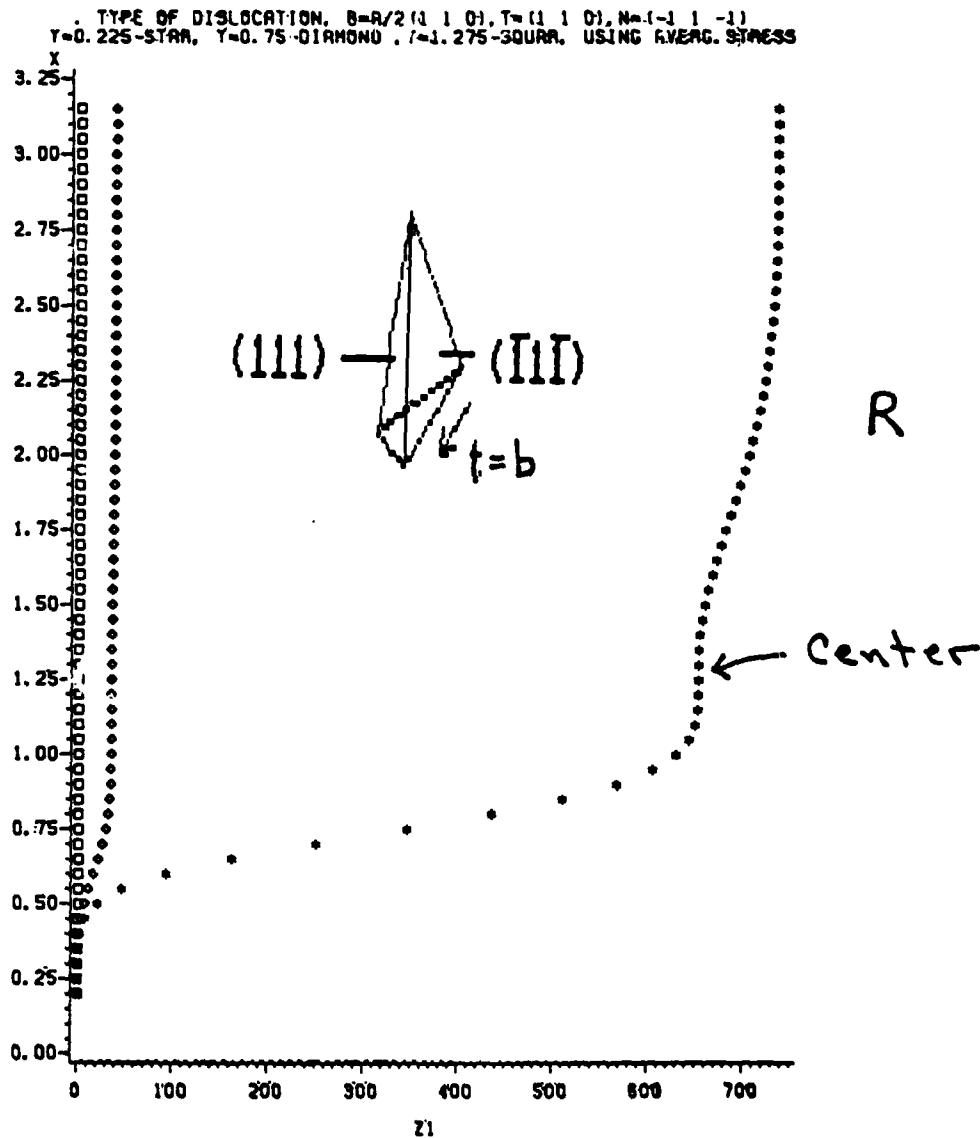
# ADVANCED SILICON SHEET

## Nine Slip Systems

burger's vector	slip plane	type of plane
$a/2 (1\ 0\ 1)$	$(-1\ -1\ 1)$	left
$a/2 (0\ 1\ 1)$	$(-1\ -1\ 1)$	left
$a/2 (-1\ 1\ 0)$	$(-1\ -1\ 1)$	left
$a/2 (1\ 0\ -1)$	$(-1\ 1\ -1)$	right
$a/2 (1\ 1\ 0)$	$(-1\ 1\ -1)$	right
$a/2 (0\ 1\ 1)$	$(-1\ 1\ -1)$	right
$a/2 (0\ -1\ 1)$	$(1\ -1\ -1)$	transverse
$a/2 (1\ 0\ 1)$	$(1\ -1\ -1)$	transverse
$a/2 (1\ 1\ 0)$	$(1\ -1\ -1)$	transverse

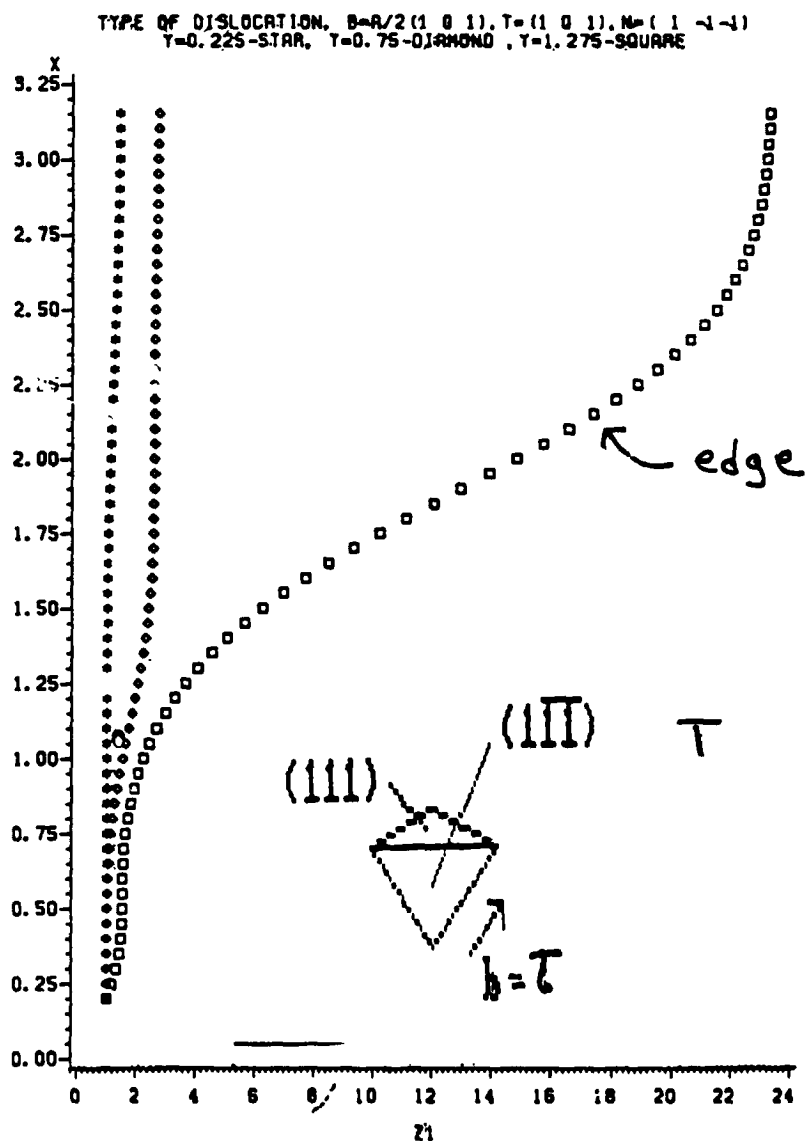
# ADVANCED SILICON SHEET

Density of Dislocations When NO is 1 at  $x = 0.2$  cm



# ADVANCED SILICON SHEET

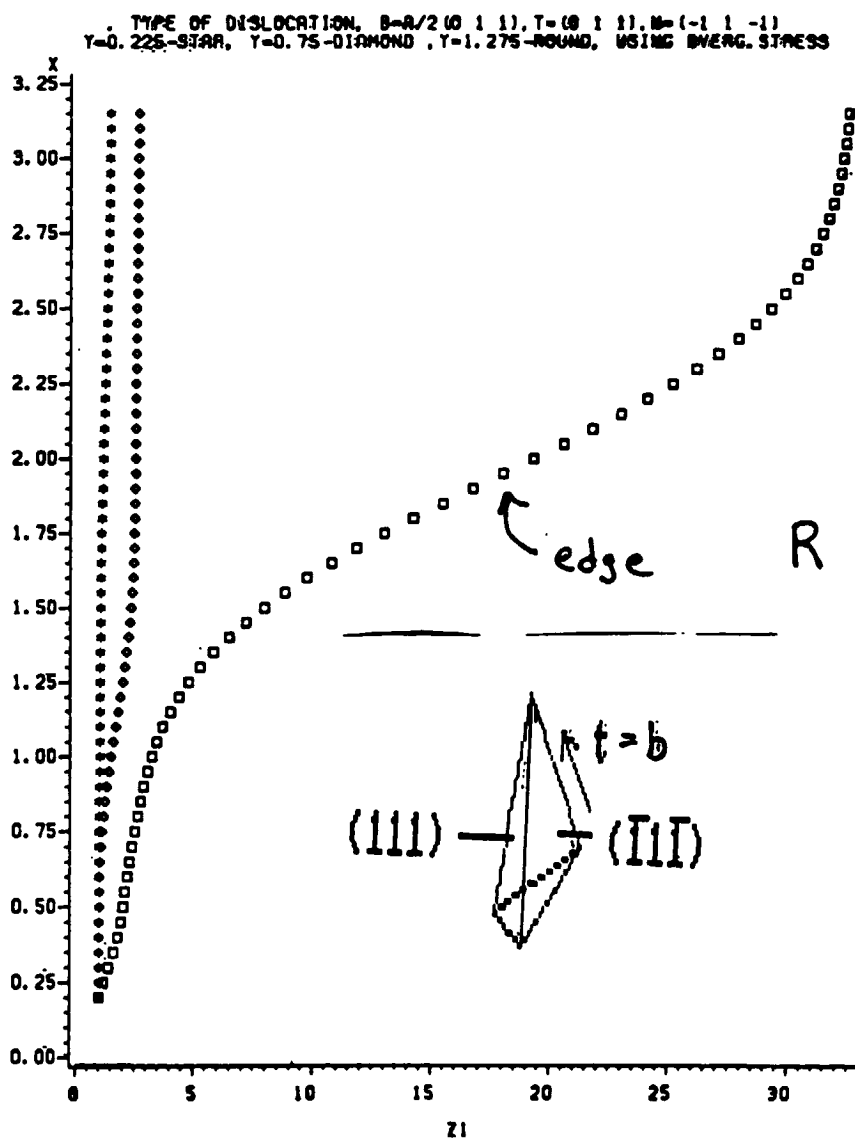
Density of Dislocations When NO is 1 at  $x = 0.2$  cm





# ADVANCED SILICON SHEET

Density of Dislocations When NO is 1 at  $x = 0.2$  cm

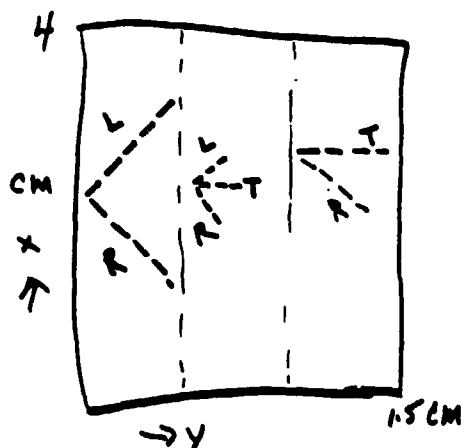


# ADVANCED SILICON SHEET

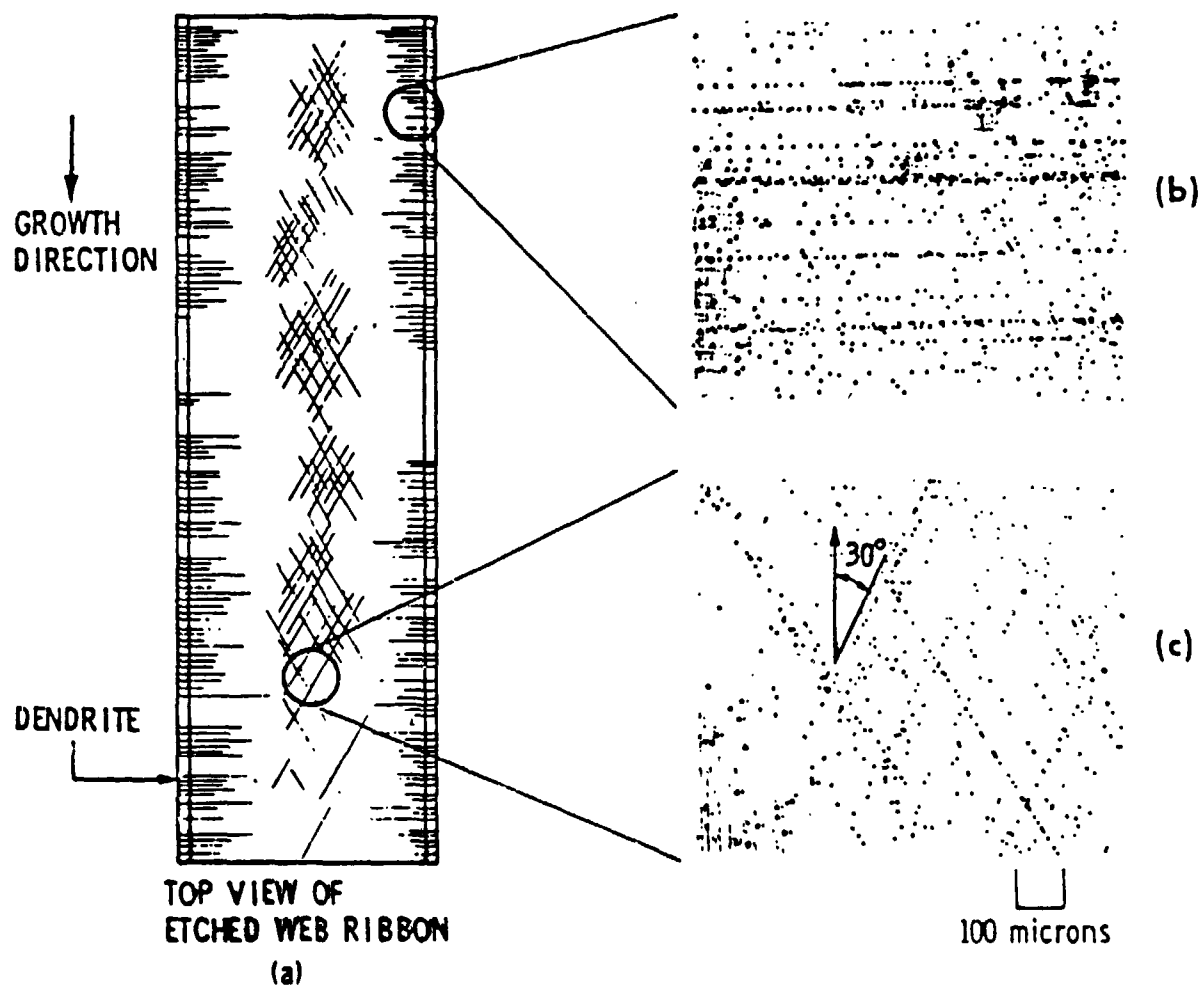
Dislocation Multiplication by Stress Averaging Over 0.5 cm

Dislocation Type			Multiplier Factor		
<u>b</u>	<u>N</u>	<u>Type</u>	<u>Center</u>	<u>Mid-Width</u>	<u>Edge</u>
[101]	[111]	L	750	50	1
[011]	[111]	L	2.5	3.5	5
[110]	[111]	L	23	1	2
[110]	[111]	R	750	50	1
[101]	[111]	R	12	1	50
[011]	[111]	R	1	2.5	35
[101]	[111]	T	1	3	24
[110]	[111]	T	2	3	10
[011]	[111]	T	1.5	66	37

Calculations started at  $x=0.2$  cm.

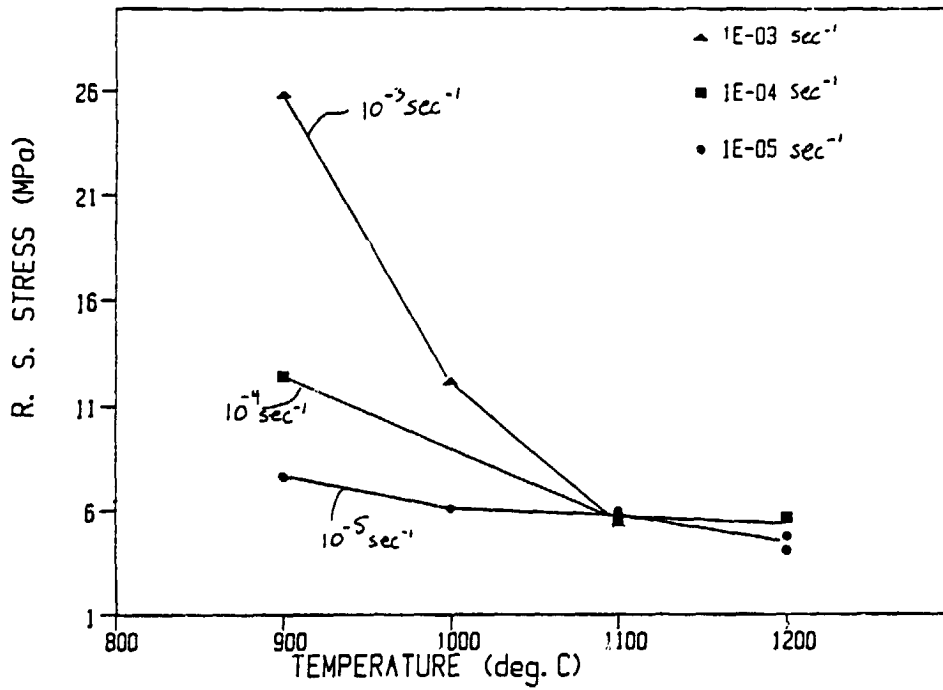


Dislocation Distribution in Web Dendrite Ribbon

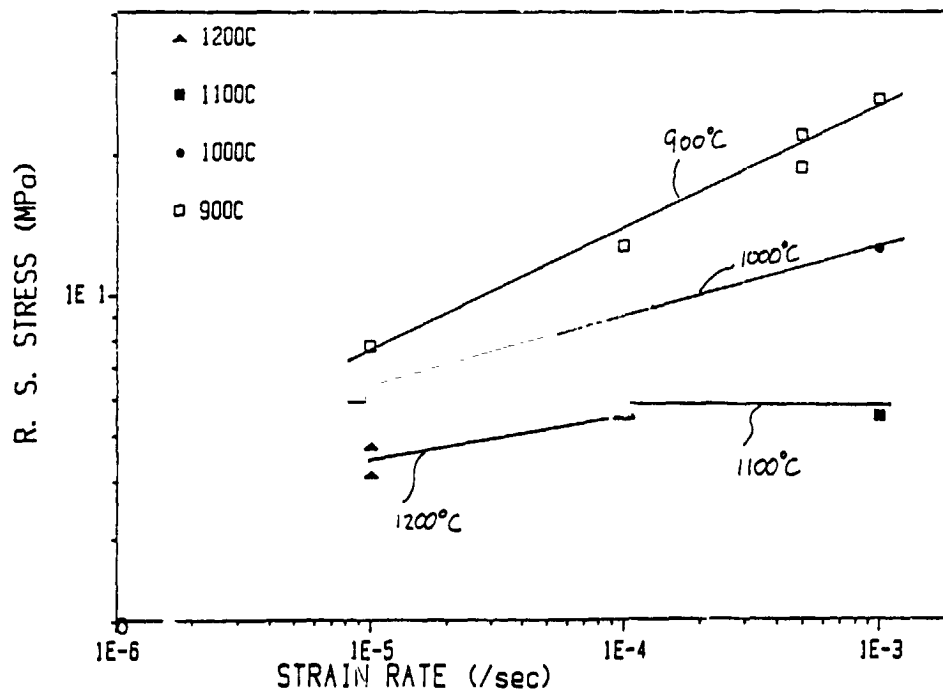


# ADVANCED SILICON SHEET

## Cz Temperature Dependence

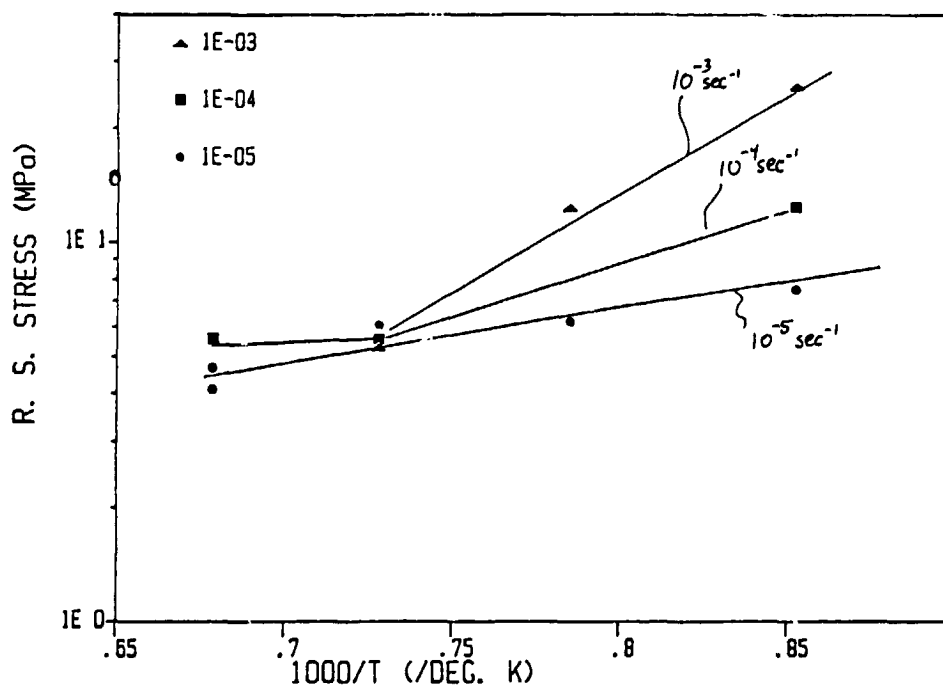


## Cz Strain Rate Dependence

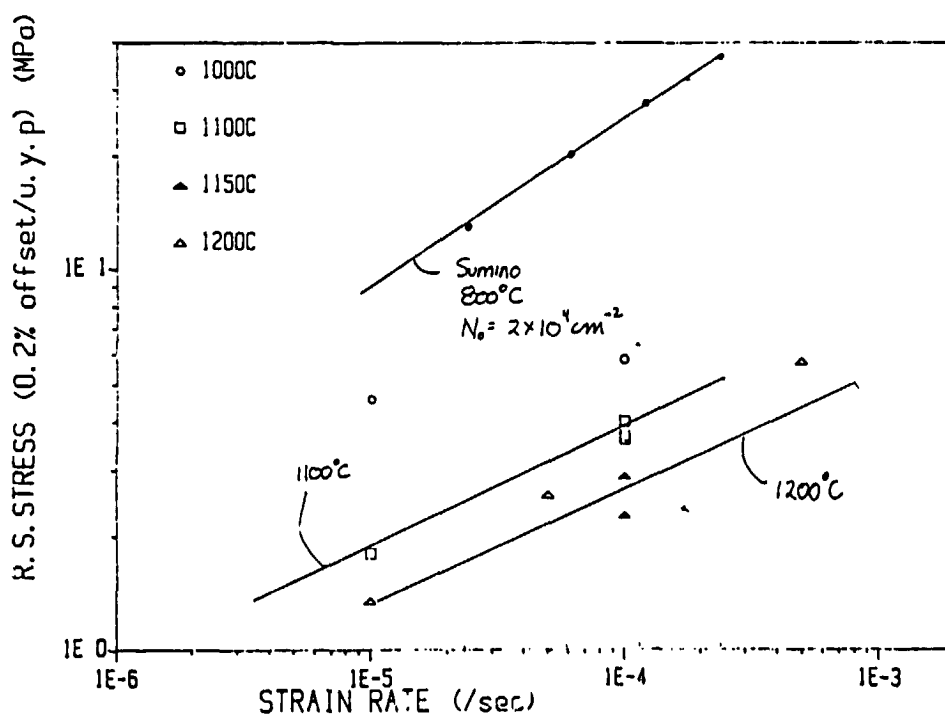


# ADVANCED SILICON SHEET

## Cz Temperature Dependence



## Web Ribbon: Strain Rate Dependence



Web Ribbon: Temperature Dependence

